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ABSTRACT

If labor market phenomena are interpreted from an allocational point of view, where individuals differing in levels of various capabilities have to be matched with jobs differing in job requirements, education can be seen as an intermediary institution affecting the capability endowment of individuals upon entering the labor market. Vertical Sorting is a situation where initially abler individuals take longer educations. Horizontal Sorting is a situation where the selected type of education equals the individual's type of capability endowment. Conditions for Vertical and Horizontal Sorting are studied, both in an open smooth labor market, and in a labor market with constraints on job choice. The conclusion is drawn that both Vertical and Horizontal Sorting work out to increase already existing inequalities between individuals. Also, even in a perfect labor market, rates of return to education will only be equalized within particular segments of the curriculum (and capability) space, but not across such segments. (Four graphs illustrate the discussion. Contains 28 references.) (Author)

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Institute for Research on Educational Finance and Governance

SCHOOL OF EDUCATION STANFORD UNIVERSITY

Program Report No. 84-B8

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Joop Hartog

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INDIVIDUAL SCHOOLING DECISIONS AND LABOR MARKET ALLOCATION: VERTICAL AND HORIZONTAL SORTING

Joop Hartog

July 1984

Joop Hartog is Professor of Economics at the University of Amsterdam, The Netherlands. Supported by a Fulbright Scholarship, he was a Visiting Scholar at the Institute for Research on Educational Finance and Governance, School of Education, Stanford University during the Spring and Summer of 1984. The analyses and conclusions of this paper do not necessarily reflect the views or policies of this organization.

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Abstract

If labor market phenomena are interpreted from an allocational point of view, where individuals differing in levels of various capabilities have to be matched with jobs differing in job requirements, education can be seen as an intermediary institution affecting the capability endowment of individuals upon entering the labor market. Vertical Sorting is a situation where initially abler individuals take longer educations. Horizontal Sorting is a situation where the selected type of education equals the individual's type of capability endowment. Conditions for Vertical and Horizontal Sorting are studied, both in an open smooth labor market, and in a labor market with constraints on job choice. The conclusion is drawn that both Vertical and Horizontal Sorting work out to increase already existing inequalities between individuals. Also, even in a perfect labor market, rates of return to education will only be equalized within particular segments of the curriculum (and capability) space, but not across such segments.

Acknowledgment

I am grateful to Henry Levin for valuable discussions and advice.

1. Introduction

An economic, neoclassical, way of interpreting the labor market is to stress the allocation problem that has to be solved there. Workers differ in abilities and skills, and hence in potential productivity. Jobs differ in the degree of complexity and difficulty of the activities to be performed, and hence in the extent to which human capabilities can be usefully employed. The neoclassical approach emphasizes that wages can be instrumental in this problem of assigning different workers to different jobs. Models of this sort have been pioneered by Tinbergen (1956) and elaborated by Sattinger (1975) and Hartog (1981a). Developments in labor economics of the last decade have stressed that the allocation problem is strongly affected by the nature of information about job requirements and worker capabilities. This aspect was first put forward by Spence (1974).

The economics of educational choice relate an individual's demand for education to the expected labor market rewards that come with extended schooling. The dominant model in use here is the human capital model, which reduces educational choice to the comparison of monetary cost and monetary returns.[1] This implies that the entire heterogeneity of the schooling process is reduced to one dimension only, the cost of acquiring the skills (or the informational badge) of a particular type of education. Differences in type and length of education are only relevant to the extent that they produce different rates of return to the investment. There is little, if any, attention for the underlying variables in the schooling process, that is for direct measures of the output of schools. Yet, the debate between human capital and sorting models precisely turns on this point. The models are similar in terms of explaining individual's demand

for education, but differ radically in the assumption on the output of the school: human capital models implicitly assume skill production, sorting models assume production of information on the individual's capabilities.

Skipping over the actual content of the schooling process is both the strength and the weakness of these models. It's a strength (and a challenge) on methodological grounds. If a problem can be solved at a higher level of abstraction, involving less variables and a more general model structure, this gives a lead over models that have to spell out many detailed relations between many variables. The crucial question then is, whether the problem actually can be solved. It's a weakness, precisely because much informational detail is lost, about differences between individuals, between types of schools and curriculum, about the association between these sets of variables and about the relation with the heterogeneous structure of the labor market.

This paper will consider some details of the educational process. Its main purpose will be to develop a model of long-run labor supply that fits in with the interpretation of the labor market given in the first paragraph. In a short-run setting, individuals with given different capabilities have to be matched with jobs that differ in their given scope for utilizing these capabilities (see Hartog, 1978). Since individuals can affect the endowments they bring to the labor market through education, this is a natural extension to consider.[2]

The model will be applied to study the role of education in the structure of capability supply. The models referred to above, on labor market allocation, naturally drift to investigation of the association between capability and job requirements: will abler individuals be found in

more difficult jobs, will there be an association between type of capability and type of job? Some models indeed conclude to such an association, e.g. Sattinger (1975), MacDonald and Markusen (1982). It is then of some relevance to know what schooling means in such a context. If, under certain conditions, the labor market sends individuals to jobs on the basis of level and type of capability, it is useful to know whether schooling also fits in: will schooling serve to develop the capabilities in which individuals have a comparative advantage, will it enlarge upon already existing differences, and if so, under what conditions will this happen? This paper attempts to answer some of these questions. It introduces a model that takes the educational production function into account and then applies it to the questions mentioned above. Prior to that, introduction and discussion of basic concepts will be necessary.

2. Concepts Used.

The concepts used here are to a large extent borrowed from occupational and educational psychology, and will be structured along the economist's distinction of supply and demand side.[3] On the supply side, psychologists have used four main categories of variables to describe differences among individuals: cognitive abilities, psychomotor abilities, personality variables and vocational preferences. The first three sets will be taken to determine an individual's potential productivity.[4] As to cognitive ability, there has been much debate about the decomposition. Spearman introduced, in 1904, the idea of a single general ability factor, and many specific factors g_i. Thorndike later denied the existence of the g-factor, and only recognized the specific factors. Thurstone's group-factor theory claims that intelligence is made up of six to ten

primary or group-factors, such as "number", "verbal", "space", "word fluency", "reasoning", and "rote memory". Recent research suggests that a limited number of cognitive abilities, similar to those mentioned by Thurstone, are indeed adequate. In the field of psychomotor abilities and physical proficiencies, the Peterson and Bownas survey, searching for the number of dimensions in the labor allocation problem, concludes to some 18 abilities, referring to various kinds of physical strength, flexibility, reaction time, dexterity and control. Personality variables are used to describe an individual's interpersonal orientation, the way he perceives himself and behaves among other individuals. Peterson and Bownas, evaluating recent research, conclude that a list of 15 variables appears most useful. The list contains such variables as sociability, impulsiveness and persistence.

Structuring the demand side of the labor market starts from the job and then attempts to reduce dimensions by looking at job content. One method is due to McCormick (McCormick et al, 1972). It determines the activities that workers have to engage in and then searches for a limited number of basic dimensions in the space of activities. A second method attempts to classify jobs according to levels of complexity of the activities in dealing with data, people and things. This method has found its way into the Dictionary of Occupational Titles, a long-time project of the U.S. Department of Labor.

Classification of jobs is usually undertaken with the purpose of applying it to the allocation problem. After grading jobs, the next step is to study the relation between individual abilities and job grades. This acknowledges the notion that a given ability may have a different value in

different jobs. The step towards the study of optimum assignment rules is then easily made,[5] and this leads to the notion of job requirements: the levels of variables that should characterize the worker selected for the job. The optimum could be a full economic optimum, taking into account cost and returns of differently qualified workers, simultaneously for all jobs in the organization. Instead of optimizing, satisficing could also be used; that would determine what kind of individual, in the given job, in the given circumstances, would come up with "average, satisfactory performance".

Measurement of job requirements typically occurs rather infrequently and adjustment to changed circumstances is not immediate. The optimizing or satisficing interpretation of job requirements can therefore at best only hold at the moment of measurement. The optimizing interpretation also suffers from another drawback: all assignments deviating from the job requirements would immediately qualify as suboptimal. In case of satisficing, deviant assignments would either be unsatisfactory (in case of underqualification) or inefficient (in case of overqualification). It would seem that such verdicts should be based on more information on the actual productivity impact of such assignments and should be postponed until more empirical work has been done.

Given the problems associated with taking job requirements as specifying strictly required worker characteristics, a different interpretation will be favored here. Job requirements will simply be taken as grading of jobs on the particular ability involved. The grade will indicate the level of complexity or difficulty of the job's activities involving the particular ability. This will ignore the optimum connotation

of the job requirement and make the interpretation similar to that of the measurement of complexities in relation to data, people and things mentioned above. One can then use these job requirements to formulate specific hypotheses. A very general hypothesis would be to state that the value of particular individual ability levels will differ for different grades of labor. More specifically, one might assume that a given ability level, c.p., leads to a higher marginal product at higher grades of labor (higher job requirements). One may also make the assumption of comparative advantage: at higher levels of job requirement, the productivity advantage of higher ability levels, relative to lower ability levels, is larger.[6]

This brief survey serves to document a particular interpretation of the labor market. Clearly, individuals differ in many ways. Such differences can be measured through the use of test scores and much information is available (for an overview and further references, see Dunnette and Fleischman, 1982). All the variables used to describe and measure these variables will be called capabilities: those characteristics of an individual that determine potential productivity. Differences among jobs, in the extent to which individuals can usefully employ their capabilities are also manifold (the most extended empirical data base is the U.S. Dictionary of Occupational Titles; for a critical appraisal, see Miller et al., 1980). All the variables used to describe jobs will be called job requirements: the level of complexity or difficulty of the job's activities involving specific capabilities.

A number of authors have dealt with the problem of allocating heterogeneous workers to heterogeneous jobs, and derived implications of this approach (Roy, 1951; Tinbergen, 1956; Mandelbrot, 1962; Sattinger,

1975; Rosen, 1978; Hartog, 1981a; MacDonald and Markusen, 1982). In these models, the different endowments of individuals are taken as given. Schooling however, can be viewed as the intermediate step in matching workers to jobs, transforming individuals' initial endowments into those useful in the labor market. Economists have turned their attention to this approach, and since the 1960s a large literature has developed on educational production functions. In the specification given by Hanushek (1972), and Levin (1976), a vector of educational outputs is derived from inputs like family background, peer group variables, school and community environment variables, a vector of innate student endowments, and a vector of initial achievement. This specification contains the aspect stressed here: schooling turns a vector of initial capability levels into a vector of output capability levels.

Conceptually, there is general agreement that education is a multi-dimensional process. Empirically, most work is limited to a single output variable, a standardized achievement test score (for surveys, see Hanushek, 1979; Lau, 1979). To some extent this is due to data problems. While for cognitive outputs (such as verbal or mathematical ability, or specific subject matter knowledge) many measures are available, such as school grades and standardized achievement scores (Hanushek, 1972, p. 21), this is not the case with non-cognitive variables. In the latter domain, there is no consensus on what to measure and how to measure it. More work on educational production functions is quite welcome, and it is easy to agree with Hanushek's (1979) conclusion that a primary gap in knowledge exists concerning the link between the school as a production process and the individual's performance in the labor market (p. 378). The screening

debate is only an illustration of that point. In the models presented below, some analysis will be given of that relation.

3. The Smooth Labor Market.

Imagine an economy where individuals act as earnings maximizers. The individuals differ in capability levels, measured by their vector of pre-school endowments a_o. There is full information on jobs and individuals, and the labor market consists of a set of jobs such that individuals can supply any capability combination $a_{oj} \geq o$ they want. Provided additional capability levels always make a positive contribution to earnings, an individual will select the job that fully utilizes all his capabilities: capability supply, without schooling, equals capability endowment.

There exist many different types of education e, which differ in prescribed length s. A type of education is not necessarily confined to a particular type of school: it also covers differences in curriculum within a type of school.[7] Educational choice will be analyzed in two steps, first selecting the type of education e for given lengths and next, selecting the optimum length of education. To facilitate analysis, the direct cost of education (such as tuition and books) will be ignored (they may be fully subsidized). For a given length of education, educational choice involves

$$(1) \quad \max_{e} \quad w \quad \{a_o, \; e \mid s\}$$

where $w\{a_o, e \mid s\}$ indicates the maximum wage rate that can be obtained through type of education e, for an individual with given capability endowment vector a_o. In this transparent world, wages will only depend on the individual's capability levels (since these determine productivity, assumed to govern rewards) and therefore, the effect of education on wages for a given individual can be written

$$(2)\quad dw(e|s) = \sum_i da_i(e) \frac{\partial w}{\partial a_i}$$

where dw(e/s) measures the effect on wages of an education e, constrained to take s years, compared to no education (e=0). The optimum education should maximize (2), and is thus seen to depend generally on two components:

- -the educational output da_i, derived from the educational production function
- -the wage effect $\partial w/\partial a_i$, derived from conditions in the labor market.

Solving this optimum problem will determine w_s^*, the maximum attainable earnings after s years of education.

The second step concerns choosing the length of education s. Ignoring tuition costs, assuming flat post-schooling age-earnings profiles and an infinite horizon,net present value of s years of schooling equals

$$(3)\quad N_s = \int_s^\infty w_s^* e^{-\rho t} dt = \frac{1}{\rho} w_s^* e^{-\rho s}$$

where ρ equals the individual's discount rate, assumed to be constant. An extra year of education will only be taken up of $N_{s+1} > N_s$, or

(4) $i_s \equiv \ln w^*_{s+1} - \ln w^*_s \gtrless \rho$

Considering the relation of the marginal return to education i_s to length of education s, an initial increase is certainly conceivable, but an eventual decrease seems inevitable: neither the capability increase from schooling nor the associated wage increase will rise indefinitely. In the remainder of this paper, it will be assumed that i_s is continuous and falling over a relevant range in the neighborhood of the intersection of i_s and ρ (which determines the optimum length of education). The situation is depicted below in Figure 1. An immediate consequence of the assumption is a predicted increase in desired length of education if ρ decreases, or if i_s increases.

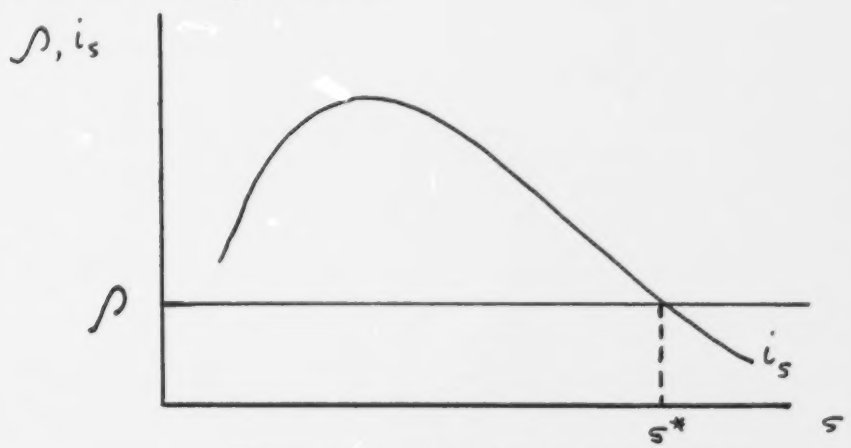

Figure 1. Optimum length of education.

The first question taken up with this model is that of Vertical Sorting: will individuals with a higher level of some capability variable a_{oi}, opt for longer education? If so, length of education would correlate positively with (initial) level of capability and schooling would

serve to enlarge capability differences between individuals. The second question that will be taken up deals with Horizontal Sorting: if individuals are distinguished by type of capability endowment, will they choose for a type of education that matches their capability endowment (in a sense to be made precise below)?

Turning to Vertical Sorting (VS) first, assume individuals have equal rate of time preference ρ and assume that the relation between i_s and s is always continuous and downward sloping around the[8] intersection with ρ. Then, if an increase in some initial capability level a_{oi} shifts the (i_s,s)-curve upwards near the point $i_s = \rho$, the new intersection will lie to the right of the old one, and hence, desired length of education increases. From (4), this is seen to require

$$(5) \qquad \frac{1}{w^*_{s+1}} \frac{\partial w^*_{s+1}}{\partial a_{oi}} - \frac{1}{w^*_s} \frac{\partial w^*_s}{\partial a_{oi}} > 0$$

The relative wage increase due to increased capability, should be higher at more extended education. Otherwise, opportunity costs will increase more than benefits. Now, since effects on type of education will be studied as Horizontal Sorting below, it will be assumed that type of education is fixed: the optimum associated with w^*_{s+1} involves the same type of education as that associated with w^*_s. Then, it is possible to write

$$(6) \qquad \frac{\partial w(a|e^*,s^*)}{\partial a_{oi}} = \sum_j \frac{\partial w}{\partial a_j} \frac{\partial a_j}{\partial a_{oi}}$$

where a_j is the capability level output after s^* years of education in type e^*, and VS is seen to require

$$(7) \qquad \frac{1}{w^*_{s+1}} \sum_j \left\{ \frac{\partial w}{\partial a_j} \frac{\partial aj}{\partial a_{oi}} \right\}_{s+1} > \frac{1}{w^*_s} \sum_j \left\{ \frac{\partial w}{\partial a_j} \frac{\partial aj}{\partial a_{oi}} \right\}_s$$

where the subscripts s+1,s refer to the point of evaluation of the derivative. Now it is clear that (7) can only hold if at least one derivative on the left-hand side is larger than its counterpart on the right-hand side. Consider the educational production function

$$(8) \qquad a = Q\{a_o, e, s\}$$

where a is the capability output vector and a_o is the input vector. Assuming

$$(9) \qquad \frac{\partial a_k}{\partial s} > 0 \quad \text{, all } k \text{ ,}$$

the evaluation of the wage component in (7) essentially comes down to the effect of higher capability levels on the wage slope, while the educational production effect can be evaluated directly. The following definitions will be used:

Wage convexity in s, i and j is defined as $\frac{\partial^2 w}{\partial aj \partial ai} > 0$.

Schooling convexity in s,i and j is defined as $\frac{\partial^2 aj}{\partial a_{oi} \partial s} > 0$.

Applying these definitions to (7), VS can only occur if there is either wage convexity in at least one (i,j) pair, or schooling convexity in at least one (i,j) pair, at the level of s corresponding to (7).[9]

Horizontal Sorting (HS) was described above as a positive association between type of capability endowment and type of education. Under what conditions will a mathematically gifted individual go into a mathematical education, while a verbally gifted individual will take up languages, say, or humanities? To analyse this question, consider the following situation. There are two types of education, A and B, of equal length. School A only improves capability A, School B only improves capability B. Let there be two individuals, differing only in these two capabilities. Individual 1 is an "A-individual", while individual 2 is a "B-individual": $a^1_{oA} > a^2_{oA}$ and $a^1_{oB} < a^2_{oB}$
(a^i_{oj} is individual i's initial endowment of capability j).

Necessary conditions for the A-individual to take up the A-education and for the B-individual to take up the B-education (HS) are[10]

$$(10) \qquad \frac{\partial w}{\partial a_A} da^1_A > \frac{\partial w}{\partial a_B} da^1_B \quad \text{and} \quad \frac{\partial w}{\partial a_A} da^2_A < \frac{\partial w}{\partial a_B} da^2_B$$

where da^i_j measures the increase in capability j if individual i completes education j. Condition (10) can be rewritten to

$$(11)\qquad \frac{\partial w/\partial a_A}{\partial w/\partial a_B} > \frac{da_B^1}{da_A^1} \quad \text{and} \quad \frac{\partial w/\partial a_A}{\partial w/\partial a_B} < \frac{da_B^2}{da_A^2}$$

where the derivatives are evaluated at each individual's point of endowment.

Assume that the educational production process is subject to <u>comparative advantage</u>: starting from $a_{oA}^1 / a_{oA}^2 > 1$, if both individuals engage in education A, the final capability ratio increases, $a_{FA}^1 / a_{FA}^2 > a_{0A}^1 / a_{oA}^2$. Similarly, starting from $a_{0B}^1 / a_{0B}^2 < 1$, if both individuals engage in education B, $a_{FB}^1 / a_{FB}^2 < a_{0B}^1 / a_{0B}^2$. Comparative advantage means that individuals relatively gifted with capability benefit most from capability j and in fact, implies that initial capability ratios will be magnified. Comparative advantage implies $da_A^1 \rangle da_A^2$ and $da_B^1 \rangle da_B^2$ and thus

$$(12)\qquad E_1 = \frac{da_B^1}{da_A^1} < \frac{da_B^2}{da_A^2} = E_2$$

E_i, i = 1,2, is the educational effect for individual i, the capability augmentation ratio of education B versus education A. From comparative advantage, this ratio is smaller for high-A individual 1 than for high-B individual 2. Condition (11) appears to require that for high-A individual 1 the relative wage slope surpasses the minimum bound E, while for high-B individual 2 it should be below upper bound E_2, and $E_1 < E_2$.

The results on HS can be illustrated graphically as follows. In Figure 2 below, individual 1's initial endowment is given by $\left\{ a_{oA}^1, a_{oB}^1 \right\}$

and individual 2's $\left\{a^2_{oA}, a^2_{oB}\right\}$. As unconstrained earnings maximizers, they would take a job where they can employ all their capabilities, as long as each of these capabilities has a positive contribution to wages. Then, for all such wage structures, the point of endowment would equal the job selected. Figure 2 gives an illustration where this occurs at a linear wage function, such that without schooling both individuals would earn the same wage.

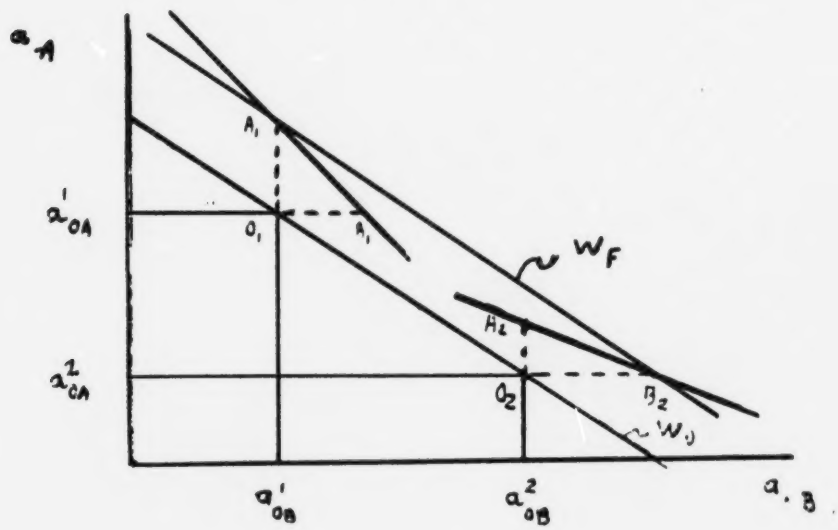

Figure 2. HS in an unconstrained labor market.

The dotted lines indicate the effect of education. School A would take individual 1 up to A_1, individual 2 up to A_2, School B would lead to points B_i, $i = 1,2$. To individual 1, school A is the best choice, as it

leads to higher earnings than school B, and the converse holds for individual 2. The condition for HS is visualized, from the comparison of the wage slope, along w_F, with the capability increase ratio: the line joining A_i, B_i for individual 1 and 2 (where the slope of the line equals V_i).

The figure illustrates an interesting case. From an initially equal earnings capacity w_o, the two individuals select a different education, following their comparative advantage, and end up with the same post-school earnings capacities w_F. In terms of the human capital analysis, they are seen to end up with the same rate of return (the HS analysis focussed on educations of given length). But this is obviously a very special case. It is quite conceivable that individual B, optimally selecting education B, reaches a higher wage level than individual A, who still optimally selects education A. Then, A has a lower rate of return than B has, and still is not led to switch to education B. Equal rates of return would require the wage function to adjust such that a wage contour w_F would go through the points A_1 and B_2, satisfying the slope conditions as in Figure 2. But in this comparative advantage model, there is no supply-adjustment mechanism to generate this equal wage condition. In equilibrium then, rates of return will differ.

It is interesting to generalize the analysis to more than one type of individual, in terms of the initial capability ratios a_{OA}/a_{OB}. Rank individuals i by the value of their relative capability endowment a^i_{oA}/a^i_{oB}, $i = 1, 2, \ldots I$. Let the capability increase ratio used above, da^i_B / da^i_A be decreasing in a^i_{OA}/a^i_{OB}. Then, if the relative wage slope $(\partial w/\partial a_A) / (\partial w/\partial a_B)$ is also expressed as a function of the relative

endowment (being the point of evaluation of relative benefits for an individual), it's the relative position of the two functions that governs the outcome. Consider Figure 3. In the situation depicted there, all individuals with relative endowment below Z have their relative wage slope smaller than the capability increase ratio, and hence, will Z choose education B, while individuals beyond choose education A. Hence, capability endowment ratio Z functions as the critical level, neatly sorting individuals with low A-endowment into the B-education and those with high A-endowment into the A-education. The same result may occur with constant or increasing wage slope ratio, but alternative configurations are conceivable. If the wage slope ratio is everywhere below the capability increase ratio, education type A does not exist, since every individual would prefer type B: developing type A capabilities is economically not viable. If the wage slope ratio is everywhere above the capability increase ratio, education type B does not exist.

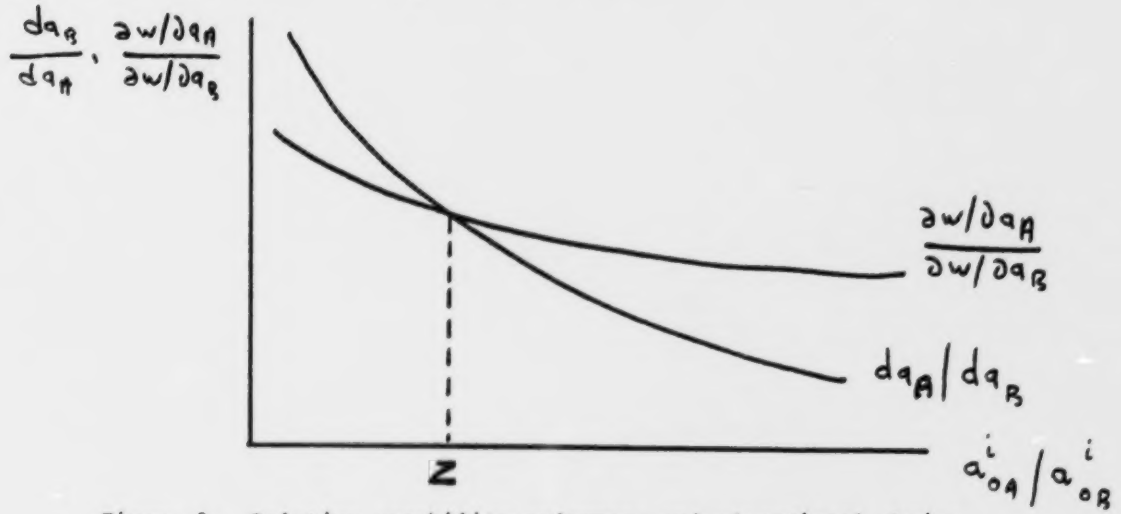

Figure 3. Relative capability endowment and educational choice.

An alternating solution is depicted in Figure 4. At low capability endowment ratios (below Z_1) and at high ratios (above Z_2), education A is chosen, while in the intermediate range, education B is selected. In this case, education A would contain a heterogeneous population: those with very low relative A-endowments and those with relatively high A-endowments.

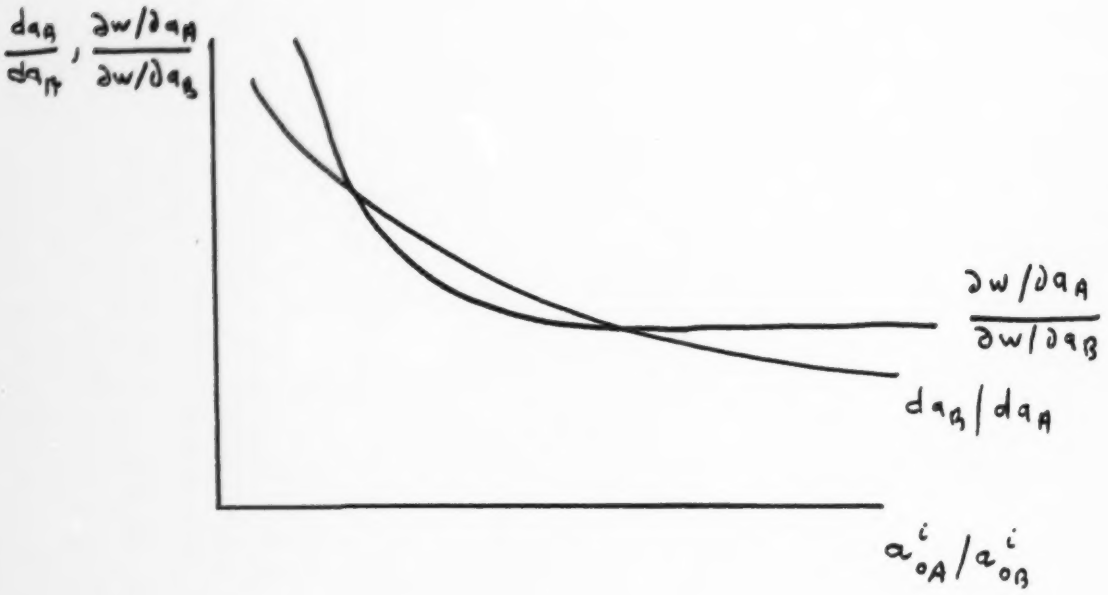

Figure 4. Heterogeneous A-schools.

Given the present analysis, the absence of such situations in reality would apparently require the assumption that the two curves have at most one point of intersection.

Another direction for generalization concerns the quality dimension. This could not meaningfully be done if quality is reduced to one dimension

only. For example, suppose individuals would be distinguished through a variable μ, measuring their relative endowment in two capabilities (e.g. mathematical versus verbal). Suppose, the schools are similarly distinguished by m, measuring the relative weight of two capabilities in the curriculum (e.g. mathematical courses versus verbal courses). Then, assuming individual values of μ and schooling values of m can take on an equal number of discrete values, HS would imply $\mu_i = m_i$, all i, that is a one-to-one relation between endowment level and schooling type. It can only reasonably be assumed that for a given level of μ_i, a schooling type characterized by a higher value of m, the output level of will be higher. Then, an individual with capability endowment μ_i can only be withheld from wanting education m_{i+1} by assuming wages to be falling, that is $w(\mu_i,m_i) > w(\mu_i,m_{i+i})$ and $w(\mu_i,m_{i-i}) < w(\mu_i,m_i)$.

There are no compelling reasons to assume that for every individual, post-school wages would be parabolic, peaking at education m_i matching endowment μ_i. HS, in such a model specification with earnings maximizers is an unlikely event. The problem essentially is a lack of degrees of freedom in the wage function. If the wage function only has a single independent variable (like μ_i), it cannot accommodate the needs of sorting.

The situation is different, if one envisages a case with n capability types, n types of individuals according to relative endowment structure and n types of education. Then, one may conceive of a wage structure in terms of "capability prices" such that the Horizontal Sorting indeed emerges.

The general structure of such a model has been laid out by Rosen (1978), in a model where workers are assigned to tasks.[11]

4. Constraints.

So far, job choice was assumed to be unconstrained: individuals could just supply all their capabilities and get rewarded for it. In reality, this may not be so easy and in this section the model will be extended to allow for constraints on the available jobs. It is now also indispensable to describe the demand side of the labor market, and this will be done through job requirements. The earnings function is changed to $w(a,r)$, reflecting the joint dependence on individual's capability level and on the job requirements. This means that the reward for a given capability level is not equal throughout the labor market, but varies according to the job that is occupied, i.e. to the extent that the capability is useful at that particular level of job requirements. In a sense, job requirements determine segments of the labor market with their own capability rewards. Empirical support for this specification is given in Hartog (1982, 1984) and Bierens and Hartog (1984). It will be assumed that $\partial w/\partial a>0$ and $\partial w/\partial r>0$.

The constraints take two forms, separate capability constraints and joint constraints. Separate capability constraints specify, for each job requirement variable, an upper bound and a lower bound within which the individual's feasible set is contained. They can be thought of as straightforward hiring standards, determining the highest and the lowest job the individual will ever be hired in. A particularly simple case, used extensively below, arises if $\bar{r}_j=a_j$: the highest level of job requirement j available to the individual is equal to his capability level. The model will start with a more general specification however. The joint

constraints bring out the fact that certain job requirement combinations are not available (for example, there may not exist a job simultaneously requiring high levels of physical strength and mathematical ability) and the notion of substitution: with a given capability endowment, it may not be feasible to obtain jobs with simultaneously high job requirements on capability k and j, but jobs scoring high on either j or k may be available.

After completing an education, the job choice problem facing an individual (characterized by capability vector a and education type e) can be represented as

$$\max_{r} w(a,r)$$

$$\text{(13)} \quad \text{s.t.} \quad \bar{h}_j\{r_j, \bar{r}_j(a,e)\} = r_j - \bar{r}_j(a,e) \leq 0 \qquad ,j=1,2,\text{---}J;$$

$$\underline{h}_j\{r_j, \underline{r}_j(a,e)\} = \underline{r}_j(a,e,) - r_j \leq 0 \qquad ,j=1,2,\text{---}J;$$

$$g_m(r,a,e) \leq 0 \qquad ,m=1,2,\text{---}M.$$

Necessary conditions for an optimum, according to the Kuhn-Tucker theorem, can be written as

$$\text{(14)} \quad \frac{\partial w}{\partial r_j} = \delta_j \frac{\partial h_j}{\partial r_j} + \Sigma_m \lambda_m \frac{\partial g_m}{\partial r_j} \qquad ,j=1,2,\text{---}J$$

h_j is taken to be either $\underline{h}_j$ or $\bar{h}_j$, depending on which constraint is binding (for given j, the upper and lower bound can only be both binding if they are equal). The multipliers measure the severity of the constraints. In an interior solution, $\partial w/\partial r_j=\delta_j=\lambda_m=0$. In case of binding constraints $\lambda_m>0$. δ_j will be positive for a binding upper bound, and will be defined negative for a binding lower bound (shifting the sign from $\partial \underline{h}_j/\partial r_j<0$ to δ_j).[12] To work out the implications of labor market constraints, two situations will be created, capability rationing and imperfect information.

4.1 Capability rationing.

In this case, information on individual capabilities is readily available. Constraints contain employers' hiring rules, setting bounds on feasible jobs for given individual capability levels. These rules derive from preset notions and a desire to prevent adjustment costs (and productivity damage) in case of misallocation. Schooling as such is not relevant in the constraints, since all that schools produce can simply be observed (from school records, grades, cheap tests). In the analyses below, often only upper bounds are assumed binding and take on a simple form:

$$h_j=\bar{h}_j=r_j-\bar{r}_j(a)=r_j-a_j \tag{15}$$

This has the convenient implication $\partial h_j/\partial r_j=1$, further reducing (14).

Educational choice can again be analyzed in two steps, first determining the type of education e that maximized earnings for a given length of education s, and then optimally selecting s. Since this latter

step is identical to the unconstrained case (apart from reformulating the assumptions on ultimately falling marginal returns to education length), only the first step will be explicitly considered. In model (13), education affects the solution by changing the parameter vector a, on which the short-run job choice problem is conditioned. The effect of such a parameter change on the objective function's maximum can be derived by applying the Envelope Theorem.[13] Then educational choice, given s, is ruled by

$$(16) \quad \max_{e} \; dw^*(e) = \sum_i \left\{\frac{\partial w}{\partial a_i} - \sum_j \delta_j \frac{\partial h_j}{\partial a_i} - \sum_m \lambda_m \frac{\partial g_m}{\partial a_i}\right\} da_i(e)$$

where $da_i(e)$ measures the increase in capability i due to education e, measured with respect to some reference type of education.[14] Now, adding assumption (15) on separate upper bounds, noting that $\partial h_j/\partial a_i = 0$, $i \neq j$ and $\partial h_j/\partial a_j = -1$, and substituting (14) for δ_j, (16) can be written as

$$(17) \quad \max_{e} dw^*(e) = \sum_i \left\{\frac{\partial w}{\partial a_i} + \frac{\partial w}{\partial r_i} - \sum_m \lambda_m \left(\frac{\partial g_m}{\partial a_i} + \frac{\partial g_m}{\partial r_i}\right)\right\} da_i(e)$$

Education is chosen to maximize the money value of capability creation, augmented by the money value of constraint lifting (evaluated at shadow prices). If there were no binding joint constraints ($\lambda_m = 0$), (17) would reduce to

$$(18) \qquad \max_{e} dw^*(e) = \sum_i \left\{\frac{\partial w}{\partial a_i} + \frac{\partial w}{\partial r_i}\right\} da_i(e)$$

In this case, the market value of the educational output $da_i(e)$ has two components: admission to a better job market segment, measured by $\partial w/\partial a_i$ (since $\partial \bar{r}_i/\partial a_i = 1$) and the return to a better paying job within a given segment, $\partial w/\partial r_i$. The effect on the joint constraints g_m similarly comes along as two partial effects.

To analyse Vertical Sorting, the same assumptions as above will be made (fixed rate of time preference ρ , falling marginal returns to education length i_s around $i_s = \rho$). The Envelope Theorem can now be applied to find the effects of changes in parameter a_{oi}, initial endowment of capability i. Simplifying h_j to simple upper bounds (15) then leads, by analogy to (7), to the following necessary conditions for VS:

$$(19) \qquad \frac{1}{w^*_{s+1}} \sum_j \left\{\frac{\partial w^*_{s+1}}{\partial a_j} + \frac{\partial w^*_{s+1}}{\partial r_j} - \sum_m \lambda_m^{s+1} \left(\frac{\partial g_m}{\partial aj} + \frac{\partial gm}{\partial rj}\right)\right\} \frac{\partial a_j^{s+1}}{\partial a_{oi}} \, da_{oi} \; > \;$$
$$\frac{1}{w^*_s} \sum_j \left\{\frac{\partial w^*_s}{\partial a_j} + \frac{\partial w^*_s}{\partial r_j} - \sum_m \lambda_m^s \left(\frac{\partial g_m}{\partial aj} + \frac{\partial gm}{\partial rj}\right)\right\} \frac{\partial a_j^s}{\partial a_{oi}} \, da_{oi} \; .$$

For a decomposition of this condition, the earlier definition of schooling convexity can be used again. Wage convexity should now be redefined:

Direct capability wage convexity in i,j is defined as $\frac{\partial^2 w}{\partial a_i \partial a_j} > 0$

Direct job requirement wage convexity in i,j is defined as $\frac{\partial^2 w}{\partial r_i \partial r_j} > 0$

Indirect wage convexity in i,j is defined as $\frac{\partial^2 w}{\partial a_i \partial r_j} > 0$

Also, an additional definition on the constraints will be necessary:

Constraint Cost m is rising in capability j, if

$$-\frac{\partial}{\partial a_j}\lambda_m\left(\frac{\partial g_m}{\partial a_j}+\frac{\partial g_m}{\partial r_j}\right) > 0$$

Then, condition (19) makes it clear that VS can only occur if at least one of the following conditions holds:

-schooling convexity in at least one (i,j) pair, at the given s

-wage convexity, either direct capability or direct job requirement or indirect, in at least one (i,j) pair

-rising constraint cost in at least one (m,j) pair.

Turning to Horizontal Sorting, the same situation that started the discussion in section 3 will be used: an A- and a B- school, an A- and a B- individual and comparative advantage. Assuming again the simple upper bound situation (15), and using (14) for δ_j, the Envelope Theorem gives a straightforward answer on necessary conditions for HS:

$$\left\{\frac{\partial w}{\partial a_A}+\frac{\partial w}{\partial r_a}-\sum_m \lambda_m^1\left(\frac{\partial g_m}{\partial a_A}+\frac{\partial g_m}{\partial r_A}\right)\right\}da_A^1 > \left\{\frac{\partial w}{\partial a_B}+\frac{\partial w}{\partial r_B}-\sum_m \lambda_m^1\left(\frac{\partial gm}{\partial a_B}+\frac{\partial gm}{\partial r_B}\right)\right\}da_B^1$$

(20)

$$\left\{\frac{\partial w}{\partial a_A}+\frac{\partial w}{\partial r_A}-\sum_m \lambda_m^2\left(\frac{\partial g_m}{\partial a_A}+\frac{\partial g_m}{\partial r_A}\right)\right\}da_A^2 < \left\{\frac{\partial w}{\partial a_B}+\frac{\partial w}{\partial r_B}-\sum_m \lambda_m^2\left(\frac{\partial gm}{\partial a_B}+\frac{\partial gm}{\partial r_B}\right)\right\}da_B^2$$

Rewriting these conditions, as in (11), would bring out the same boundaries on slope ratios E_1 and E_2, as in (12). In fact, with $\delta_j = \lambda_m^i = 0$, all i,j,m, the earlier model would reappear. If the joint constraints are not binding ($\lambda_m^i = 0$), the earlier condition on wage slopes is now seen to be

expanded to the sum of the wage effects of the school's capability output: within a given labor segment (the effect of r) and across segments (the direct effect of a). If joint constraints are also binding, the monetary value of the constraint lifting is simply added to the "price ratios "that are bound by the capability output ratio's E_1 and E_2. In the special case that upper bounds $\bar{r}_j$ are not binding, that there is only one joint constraint g and that wages are job determined (represented by the wage function w(r)), condition (20) would be reduced to a condition on constraint cost ratios. $(\partial g/\partial a_A) / (\partial g/\partial a_B)$.[15]

4.2 Imperfect information

Instead of assuming that individual capabilities can be observed immediately upon labor force entry, it will now be assumed that such observation takes time, and comes along with experience: capabilities are inferred from individuals' performance on the job. Initially, capabilities are unknown, and hence observable schooling is used to set the allocational constraints: $\bar{r}=\bar{r}(e)$; for simplicity, only upper bounds are assumed to be ever binding. In the wage function, capability levels are maintained. w(a,r) refers to lifetime earnings, and it is assumed that eventually capabilities are rewarded. However, rewarding takes place in the segment of initial assignment, which was based on schooling completed. Reallocations across segments (based on r) are assumed too costly to the worker and the firm.[16]

Again using the two-stage analysis of schooling decisions, only the choice on type of education e has to be reconsidered. According to the Envelope Theorem, educational choice follows

$$(21) \quad \max_{e} \; dw^*(e) = \sum_i \frac{\partial w}{\partial a_i} \, da_i(e) - \sum_j \delta_j \frac{\partial h_j}{\partial \bar{r}_j} \, d\bar{r}_j(e) - \sum_m \lambda_m \, dg_m(e)$$

$d\bar{r}_j(e)$ and $dg_m(e)$ represent the effect of completing education e on the feasible set of jobs.[17] With only upper bounds $\bar{h}_j$ (potentially) binding, and with $\bar{r}_j = \bar{r}_j(e)$, using (14), (21) can be written as

$$(22) \quad \max_{e} \; dw^*(e) = \sum_i \frac{\partial w}{\partial a_i} \, da_i(e) + \sum_j \frac{\partial w}{\partial r_j} \, d\bar{r}_j(e) - \sum_m \lambda_m \left\{ dg_m(e) + \sum_j \frac{\partial g_m}{\partial r_j} \, d\bar{r}_j(e) \right\}$$

Education type e is selected, for given length s, that has the largest monetary effect, both through wages and through constraint lifting. In contrast to the earlier section with full capability information however, constraint lifting is now through the effect of schooling. This precludes individual effects derived from the educational production process and makes allocational constraints identical for all graduates of a given education.

Vertical Sorting in this case will have to be brought about by the effect of capability increase through the (lifetime) wage effect. By assumption, VS is studied with type of education e fixed, and this precludes any allocation effect through the screening mechanism. Hence, VS, on the conditions used earlier (falling i_s around $i_s = \rho$), requires

$$(23) \quad \frac{1}{w^*_{s+1}} \sum_j \frac{\partial w}{\partial a_j} \frac{\partial a_j^{s+1}}{\partial a_{oi}} \, da_{oi} > \frac{1}{w^*_s} \sum_j \frac{\partial w}{\partial a_j} \frac{\partial a_j^{s}}{\partial a_{oi}} \, da_{oi}$$

Interestingly, the same condition results as in the smooth labor market case, condition (7). This is so, since constraints are not affected.

Without Spence's (1974) assumption of differential cost of education by individual's capability level, VS may occur if at least some direct capability reward effect survives in a screening environment.

Horizontal Sorting, in this model, requires conditions on the wage effects through capability augmentation and through screening, and on the value of shifts in the joint constraints as dependent on type of education. Written out for the two types of individuals who compare educations A and B, assuming the separate capability constraints to take the form $r_j - \bar{r}_j(e) \leqslant 0$, and the joint constraints to take the form $g_m(r,e) \lesseqgtr 0$, the conditions for HS are

$$\frac{\partial w}{\partial a_A} da_A^1 + \frac{\partial w}{\partial r_A} d\bar{r}_A - \sum_m \lambda_m^1 dg_{mA} > \frac{\partial w}{\partial aB} da_B^1 + \frac{\partial w}{\partial r_B} d\bar{r}_B - \sum_m \lambda_m^1 dg_{mB}$$

(24)

$$\frac{\partial w}{\partial a_A} da_A^2 + \frac{\partial w}{\partial r_A} d\bar{r}_A - \sum_m \lambda_m^2 dg_{mA} < \frac{\partial w}{\partial a_B} da_B^2 + \frac{\partial w}{\partial r_B} d\bar{r}_B - \sum_m \lambda_m^2 dg_{mB}$$

Where da_e measures the capability increase in education e for individual i, $d\bar{r}_e$ measures the shift in constraint $\bar{r}_e$ due to education e and dg_{me} measures the shift in the joint constraint due to education e. The generalization from conditions (10) in the smooth market is clear: the effects through separate and joint allocation constraints are simply added. Compared to the capability rationing case (20), constraint easing now works through the educational effect on bounds, rather than through capability increases. Sufficient (overly restrictive) conditions for HS can now be obtained by looking at the three components in the equalities. Thus, HS would obtain if wage condition (11) holds, and if two conditions

on constraint relaxing value hold. The first of these applies to the separate upper bounds:

$$(25) \qquad \left(\frac{\partial w/\partial r_A}{\partial w/\partial r_B}\right)_2 < \frac{\partial d\bar{r}_B}{\partial d\bar{r}_A} < \left(\frac{\partial w/\partial r_A}{\partial w/\partial r_B}\right)_1$$

The relative job requirement wage slope, of A for B, should be lower for the low-A individual than for the high-A individual.[18] Decreasing marginal job requirement wages would violate this condition, unless there was a sufficient strong cross-derivative of the wage function. The second condition then applies to the remaining joint constraints which cannot be rewritten any further in the general case. They come down to the condition that for individual 1, the value of joint constraint relaxation through education A should surpass that of B, while for individual 2 the converse should hold. In case of only one joint constraint, this would reduce to

$$(26) \qquad \left(\frac{dg_A}{dg_B}\right)_1 > 1 \quad , \left(\frac{dg_A}{dg_B}\right)_2 < 1$$

since the shadow-prices λ^i cancel.[19] These conditions are quite straightforward.

5. Utility maximization

Utility maximization generally is a more adequate assumption on individual behavior than earnings maximization. It is fairly

straightforward to apply that assumption in the present analysis. Since the structure of the solutions is not really affected, the full analysis will not be reproduced here; instead, a short general discussion will be given.

The utility function will be given the specification

$$U = U\{w(a,r),a,r\} \tag{27}$$

stating that jobs are evaluated on earnings and job requirements, given the individual's capability endowments. Bringing in job requirements and capability levels represents the notion that the effort that an individual has to make on the job depends on capabilities. Given the utility function and the constraints on the feasible job set, optimum conditions on job choice and on choices regarding type and length of education are easily derived. Education type is now selected for maximum contribution to utility, optimum education length is now at equality of marginal utility of extended education and rate of time preference.

Vertical Sorting requires conditions on the utility function. It was shown earlier that VS requires that the marginal benefit of higher initial endowment should be larger at longer education, i.e. larger at s+1 than at s. Marginal benefit now applies to marginal utility, and here, decreasing marginal utilities (of earnings) and increasing marginal disutilities (of efforts) can work against VS. It is not easy to obtain definite answers here, since it is not clear a priori what the structure of preferences will be. The following assumptions seem easily acceptable:

$$U_{r_i} < 0 \qquad U_{r_i r_i} < 0$$
$$U_{a_i} > 0 \qquad U_{a_i a_i} < 0$$
$$U_w > 0 \qquad U_{ww} < 0$$
$$U_{a_i r_i} > 0$$

This represents falling marginal utility (increasing marginal disutility) in a straightforward fashion. The only cross-derivative that was signed is $U_{a_i r_i}$; the positive sign indicates that a more demanding job involves less effort for an individual of higher capability level. Unambiguous results can be obtained by defining <u>diminishing marginal utility dominance</u>:

$$U_{wa_i} \lessgtr 0, \; U_{wr_i} \lessgtr 0, \text{ all } i$$
$$\sum_j \frac{\partial U}{\partial r_j \partial a_i} da_j < 0, \text{ all } i, \text{ for all } da_j \text{ resulting from an additional year of education.}$$

With diminishing marginal utility dominance, in the capability rationing case VS can only occur if there is at least some wage convexity, schooling convexity or constraint cost increase, and in the imperfect information case if there is at least some wage convexity or schooling convexity (where all these concepts were defined earlier).

<u>Horizontal Sorting</u> in the capability rationing case requires conditions very similar to those applying to earnings maximization. It's the same ratios E_1 and E_2 that set boundaries now on the ratios of original utilities obtained from higher wages, higher capability levels and less pressing constraints on job choice. Analysis of the imperfect information case also provides answers that are immediately analogous to earnings maximization. Apart from rewriting conditions (24) to the utility

metric, there is one added component, referring to the marginal utility of augmented capability levels.

6. Implications.

Vertical Sorting works out as a magnifier of inequalities between individuals. If abler individuals take up a longer education, the post-school capability distribution will be more unequal than the pre-school distribution. The magnification of inequalities is even increased by the very causes of Vertical Sorting: convexities in the wage functions or the schooling production function, or increasing constraint cost. Convexities in the wage function presumably stem from the nature of production technology and should be taken as given. Educational convexities are important problems for educational policies. As was emphasized by Brown and Saks (1975), the goals of schools may be elitist or egalitarian. Elitist school policies would favor increasing inequalities, by devoting school resources mainly to the better students; such a policy would concentrate on efficiency of resource utilization. Egalitarian school policies,by contrast, would concentrate on the equity effects of schooling and would aim at reducing inequalities between individuals. Such egalitarian policies may conceivably go so far as to reduce Vertical Sorting, thereby increasing the efficiency cost of egalitarian action. Evaluating the role of rising constraint cost in producing VS obviously requires evaluating the constraints themselves. This will depend on the feedback from observed individual performances into the constraints, to determine whether the constraints are efficient or not. As a final remark on VS, note that it may occur at one capability and at the same time not occur at another (leading to interactions with Horizontal Sorting). Casual

observation indeed suggests VS for a general ability measure like IQ or for mathematical ability (see the data in Willis and Rosen, 1979), but casual observation also suggests that it may not occur for commercial abilities: those with high levels of commercial talents may find it too expensive to remain in school.

Horizontal Sorting could only be perfect if the number of types of education equals the number of capabilities. In practice, the case would seem to be that education types outnumber capabilities.[20] By implication then, some curricula draw identical capability types while at the same time there also exists a demarcation between sets of curricula on the basis of capability types. Different curricula may be optimal for the same capability types and one may expect equilibrium to involve equal rates of return to education. Other sets of curricula draw from different capability sets and rates of returns will not be equalized between these two sets of capability types (and associated curriculum). Where the borderlines run will depend on the numbers mentioned and on the fulfillment of conditions derived above. These conditions can be taken to set limits on wage derivatives and on sensitivities of shadow prices depending on the effect of schooling on capability output and on allocational labor market constraints. If HS occurs,the post-school earnings distribution will be much different from the pre-school potential earnings distribution. Define the latter as the distribution of the individual's earnings[21] at their best job at the given wage function, then, under HS, each individual will develop that capability with which he is already relatively highly endowed, and this will expand existing inequalities. This leads to an important conclusion: if there is VS and

HS, schooling will lead to a post-schooling earnings distribution that is more unequal than the pre-school potential earnings distribution, at a given wage function. The latter qualification is important, since it is obviously conceivable that the wage function changes if participation in education changes. But that is a different problem, not studied in the present paper.

The problem that was tackled in this paper centered on the role of education in affecting the individual distribution of endowments upon entering the labor market. Above, the conjecture was made that VS and HS both tend to increase already existing inequalities in economic opportunities. The question then indeed becomes important what the empirical status of VS and HS actually is. There is some evidence that both VS and HS indeed do occur. Evidence on VS was cited above. As to HS, Kodde and Theunissen (1984) provide some evidence for the Netherlands with respect to curriculum choice.[22] But the evidence does not all go in one direction. Watson, reporting on drop-outs in the Ontario secondary school system, shows (p. 38) that 39.2% of the drop-outs had achievement levels of C or higher (where C is equivalent to 60-69%)and that over 40% (p.81) of these "C+drop-outs" left school for economic reasons (they "got a job offer" or "needed money"). Clearly, the empirical issue is quite complex and needs much careful work. Willis and Rosen (1979) provided a good start, by implementing a choice-theoretic framework similar to the one used here for self-selection with respect to length of education. The present analyses indicate that it might be beneficial to include type of education as well.

Footnotes

[1]Surely the model is general enough to include psychic cost and returns, but most applications ignore this.

[2]It is equally natural to consider the choices that firms make on job structure and job requirements. This will be taken up in a next paper.

[3]The discussion draws heavily from Peterson and Bowmas (1982).

[4]As to the fourth category, it is interesting to note in passing, that psychological research has identified a limited number of stable types of preferences. To quote from Peterson and Bowmas (1982, p. 83): "Evidence collected by many investigators...indicates that vocational preferences are highly stable (after age 21) for individuals over long periods of time (up to 25-30 years). They are also reasonably effective predictors of future occupational classification for persons--especially within broad occupational families as opposed to highly specific occupations". Perhaps not surprisingly, they have poor predictive value for performance within occupations.

[5]Hunter and Schmidt (1982) claim, on the basis of rather conservative assumptions, that U.S. output could easily rise by some 8%, if assignment of individuals to jobs were based on proper use of individuals' test scores for relevant abilities.

[6]Hunter and Schmidt (1982) give some empirical evidence that the dispersion of the dollar value of performance across individuals differs among occupations. Comparative advantage is a prominent feature in related work by Sattinger (1975), Rosen (1978) and MacDonald and Markusen (1982).

[7] Just to illustrate the tremendous size of the choice set: Kodde and Theunissen (1984), in an analysis of educational careers in the Netherlands, note that after secondary education, individuals can choose among at least 130 types of education in higher education (p. 118).

[8] It is assumed that there is only one intersection that also satisfies the second-order conditions for an earnings maximum.

[9] The decomposition of (7) into two convexity conditions only makes sense if one takes the scale of measurement of capability levels as a useful reference. Test scores should neither be standardized for age nor for education.

[10] It will be assumed that for each individual both educations yield a sufficient rate of return.

[11] The analogy of the present model to that of Rosen is quite close and the results obtained above for the case of individuals ranked by a_{oA}/a_{oB} are very similar to his results of assigning different workers to identical tasks (section III).

[12] See Varian (1978), p. 259. It is assumed that "constraint qualification" holds for the binding constraints.

[13] Varian (1978), p. 268.

[14] In particular for lengthy education, it may be best to measure the increases relative to some reference education, to make the evaluation with first-order derivatives more adequate: it would not be apt to measure these derivatives at zero-years of education.

[15] This result is found from setting $\delta_j=0$ in the general school effect equation $\{ \partial w/\partial a_e - \delta_j \partial h_j/\partial a_e - \Sigma\lambda_m \delta g_m/\delta g_a \}$, and bringing in the other conditions mentioned in the text ($M=1, \partial w/\partial a_e = 0$).

[16] A model with initial allocation based on schooling, but without transfer cost, is developed in Hartog (1981b). Empirical support for the assumptions made here is presented in Hartog (1983). Taubman (1975) shows that the effect of ability on earnings is stronger at more advanced stages of the career. Note that the present analysis focuses on labor supply behavior, and is not an equilibrium model of the labor market.

[17] A simple specification of $dg_m(e)$ might occur with $g_m(r,e) = g_m^1(r,e) - \alpha_m \leq 0$ and e only affecting the intercept, i.e. $dg_m(e) = -d\alpha_m(e)$.

[18] Note that the two conditions now have a joint interval boundary because dr_B/dr_A is not differentiated by individuals.

[19] Since $\lambda_m > 0$, and since it is supposed that constraint relaxing has a positive effect, $-dg_{me}$ is taken to be positive. For an illustration, see note 17.

[20] From the survey in section 2, one may conclude that a number of 50 capabilities would most certainly be an upper bound; note 7 at the beginning of Section 3 cites the existence of 130 types of higher education curricula.

[21] In case of utility maximization, the discussion should be cast in terms of the distribution of welfare rather than of earnings.

[22] In secondary school, students choose Latin and Greek significantly more often if in elementary school they scored high at language; they choose sciences significantly more often if in elementary school they scored high at mathematics. Similar results hold on type of secondary school chosen.

References

Bierens, H. J. and J. Hartog (1984). On specifying, estimating and testing regression models with discrete explanatory variables, with an empirical application to the earnings function. University of Amsterdam, Department of Economics.

Brown, B. W. and D. H. Saks. The production and distribution of cognitive skills within schools. Journal of Political Economy. 83(3),571-594.

Dunnette, M.D. and E.A. Fleishman (1982). Human capability assessment, Volume 1 of: Human performance and productivity, edited by E.A. Fleishman. Hillsdale, N,J.: Lawrence Erlbaum.

Hanushek, E. A. (1972). Education and race. Lexington, MA: Lexington Books.

____ (1979). Conceptual and empirical issues in the estimation of educational production functions. Journal of Human Resources XIV (3), 351-388.

Hartog, J. (1978). On the multicapability theory of income distribution. European Economic Review, 10(2), 151-171.

____ (1981a). Personal income distribution: a multicapability theory. Boston, MA: Martinus Nyhoff.

____ (1981b). Wages and allocation under imperfect information. De Economist, 129(3), 311-323.

____ (1982). Allocation and the earnings function. Research Memorandum 8205, University of Amsterdam, Department of Economics.

____ (1983). To graduate or not: does it matter? Economics Letters, 12, 193-199.

_____ (1984). Earnings functions: beyond human capital. Research Memorandum 8413, University of Amsterdam, Department of Economics.

Hunter, T. E. and F. L. Schmidt (1982). Fitting people to jobs: the impact of personnel selection on national productivity, in Dunnette and Fleishman (1982).

Kodde, D. A. en M.A.M. Theunissen (1984). De schoolloopbaan van de snelle VWO-leerlingen, Economisch Instituut, Katholieke Universiteit Nymegen.

Lau, L. J. (1979). Educational production functions, in Economic dimensions of education, Washington,D.C.: National Academy of Education.

Levin, H.M. (1976). Concepts of economic efficiency and educational production, in : J. T. Froomkin, D. T. Jamison, R. Radner (eds.), Education as an industry. Cambridge, MA: Ballinger for NBER.

MacDonald, G. M. and T. R. Markusen (1982). The role of skills and technology in the determination of optimal assignments within the firm. Research Report 8211, University of Western Ontario, Department of Economics.

Mandelbrot, B. (1962). Paretion distributions and income maximization, Quarterly Journal of Economics, 76, 57-85.

McCormick, E. J., P. R. Jeanneret and R. C. Mecham. A study of job characteristics and job dimensions as based on the Position Analysis Questionnaire (PAQ), Journal of Applied Psychology Monograph 56 (4), 347-68.

Peterson, N. G. and D. A. Bownas (1982). Skills, task structure and performance acquisition, in Dunnette and Fleishman (1982).

Rosen, S. (1978). Substitution and division of labor, Economica 45, 235-50.

Roy, A. D. (1951). Some thoughts on the distribution of earnings, Oxford Economic Papers, 3, 135-46.

Sattinger, M. (1975). Comparative advantage and the distributions of earnings and abilities, Econometrica, 43(3), 455-68.

Spence, A. M. (1974). Market signaling, Cambridge, MA: Harvard University Press.

Taubman, P. J. (1975). Sources of inequality in earnings. Amsterdam: North-Holland.

Tinbergen, J. (1956). On the theory of income distribution , Weltwirtschaftliches Archiv, 77, 156-75.

Varian, H. R. (1978). Microeconomic analysis, New York: Norton.

Watson, C. Focus on dropouts, Toronto: Ontario Institute for Studies in Education.

Willis, R. J. and S. Rosen (1979). Education and self-selection, Journal of Political Economy, 87 (5 part 2), S7-S36.